

# Multinomial Logistic Regression for Arrhythmia Detection

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# Summary

- We classify 48683 heartbeats of 29 patients into 4 classes
  - normal, arterial contraction, junctional contraction and ventricular contraction beats
- We adapt Multinomial Logistic Regression (MLR) as a classifier of cardiac arrhythmia
  - It learns the posterior probability distributions of each class
  - It is applied to arrhythmia detections

# Medical Context & Motivation

- Electrocardiogram (ECG):
  - reflects activity of the central blood circulatory system.
  - provides information on the normal or pathological physiology of heart activity.
  - is an important non-invasive clinical tool for the diagnosis of **heart diseases**.
- Early and quick detection and classification of ECG arrhythmia is important, especially for the treatment of patients in the intensive care.

# Medical Context

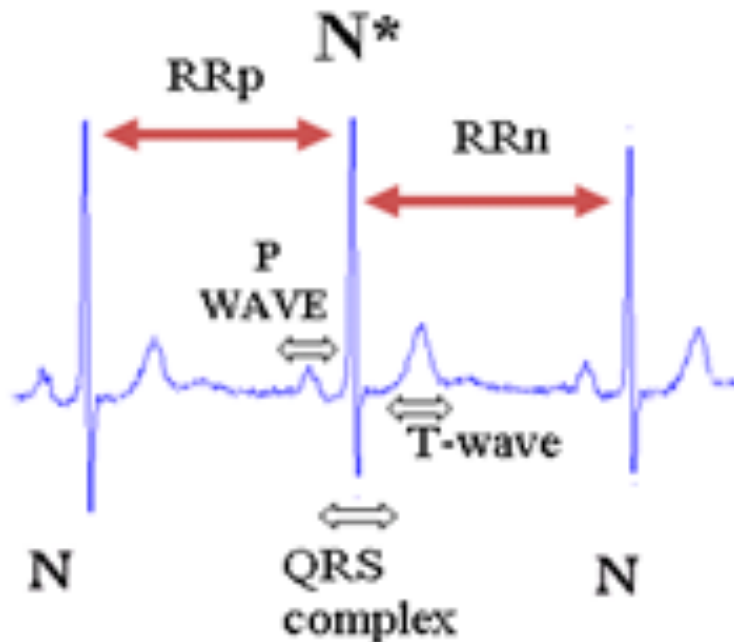
- Computer aided diagnostic (CAD) systems have been used for ECG classification.
- Popular techniques:
  - Multivariate statistics, decision trees, fuzzy logic, expert systems and hybrid approaches
- The most important step is the integration of suitable **feature extractor** and **pattern classifier**.

# Data

- 29 patients of the MIT-BIH db. Conditions:
  - ventricular contraction beats (PVC)
  - premature arterial contraction beats (PAC)
  - premature junctional contraction beats (PJC)
- 48683 heartbeat samples

Beats			
N: normal	A: arterial contraction	J: junctional contraction	V: ventricular contraction
43943	711	32	3997
90.26%	1.46%	0.07%	8.21%

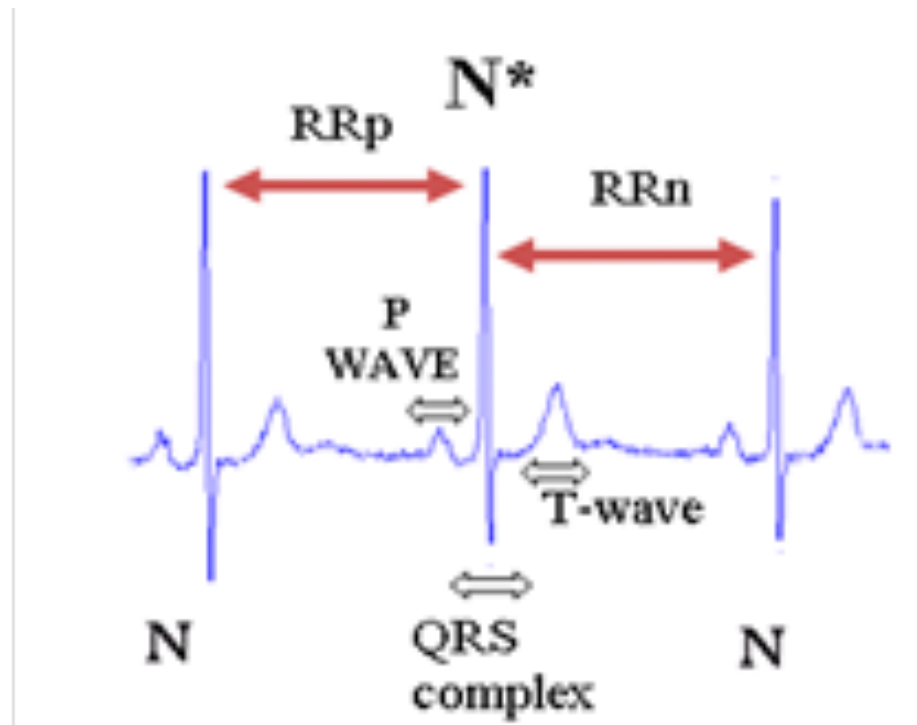
# Example of ECG Beat



Attributes	Meaning
RR precedent: RR0	Distance between the peak of the current beat R and the previous one
RR next : RRn	Distance between the peak of the present R and the next beat
QRS complex	Beginning of the Q-wave and the end of the S wave
Comp	The ratio $RR0/RRs$
PP	Peak to peak of the R wave of the QRS complex
Energy	Energy of the QRS complex

# Data Preparation

- R peaks were identified by the Tompkins algorithm

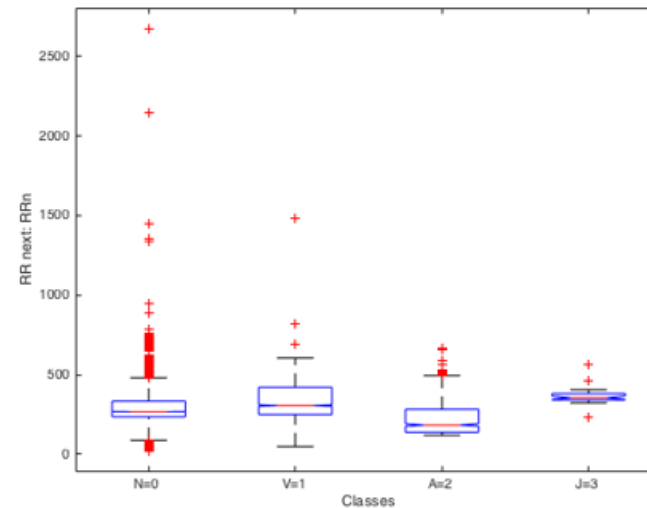
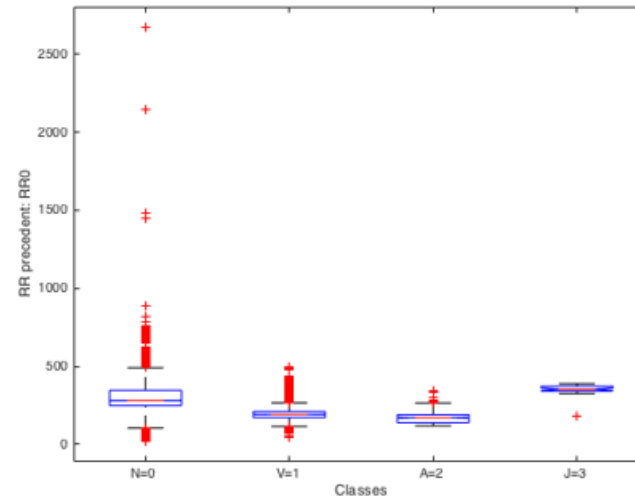


# A non-trivial classification task

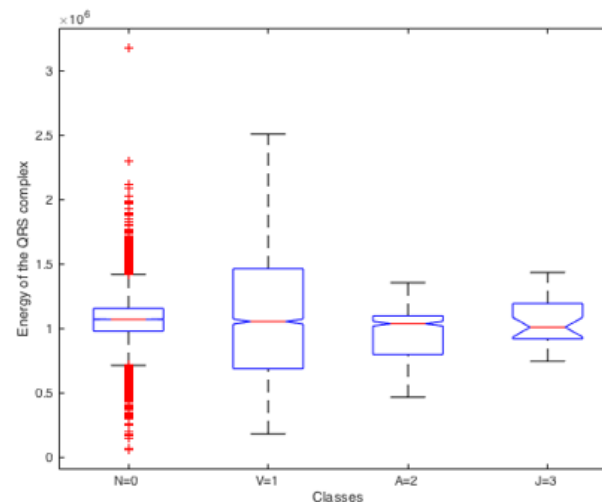
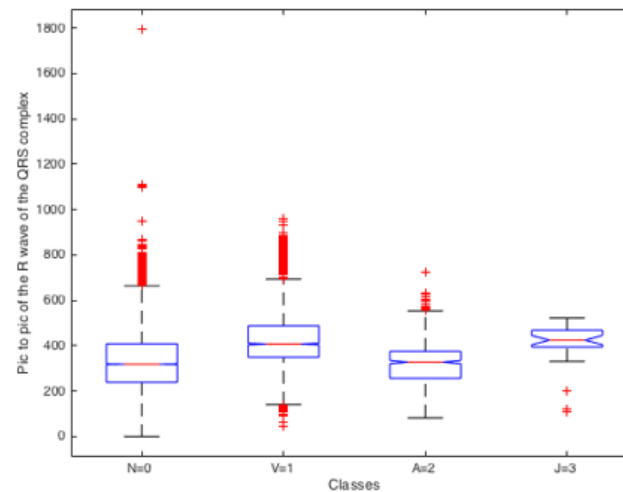
- Classifying heartbeats with only one feature, e.g. RR0, RRS, QRS, etc. is complex due to the lack of specific threshold for each class.



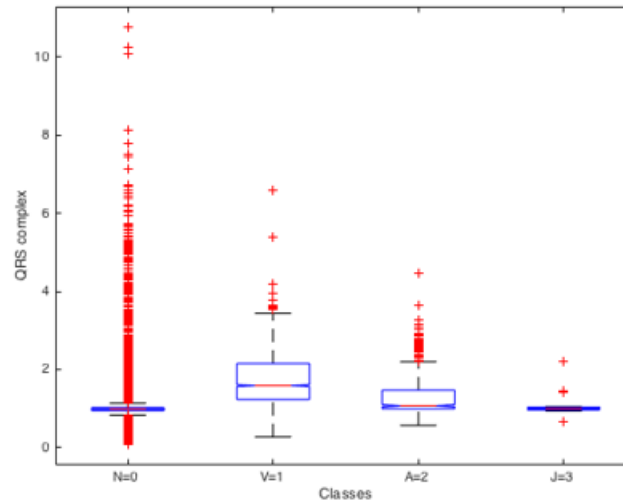
# Distribution of $RR_0$ and $RR_n$



# Distribution of features PP and ENERGY of QRS complex in different classes

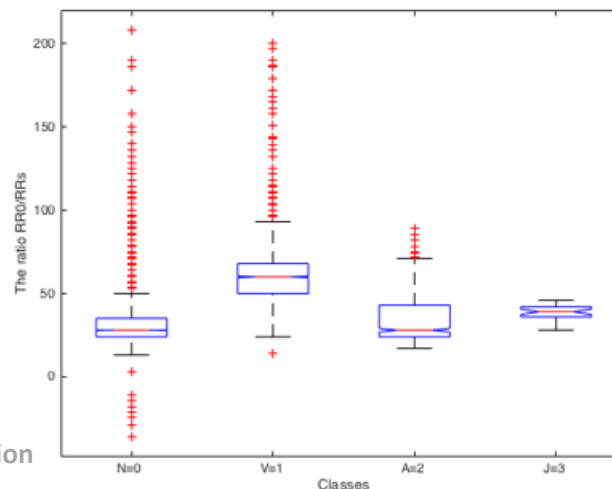


# The distribution of features QRS and COMP in different classes



➔ We can separate between V and N classes

➔ This is exploited in our approach

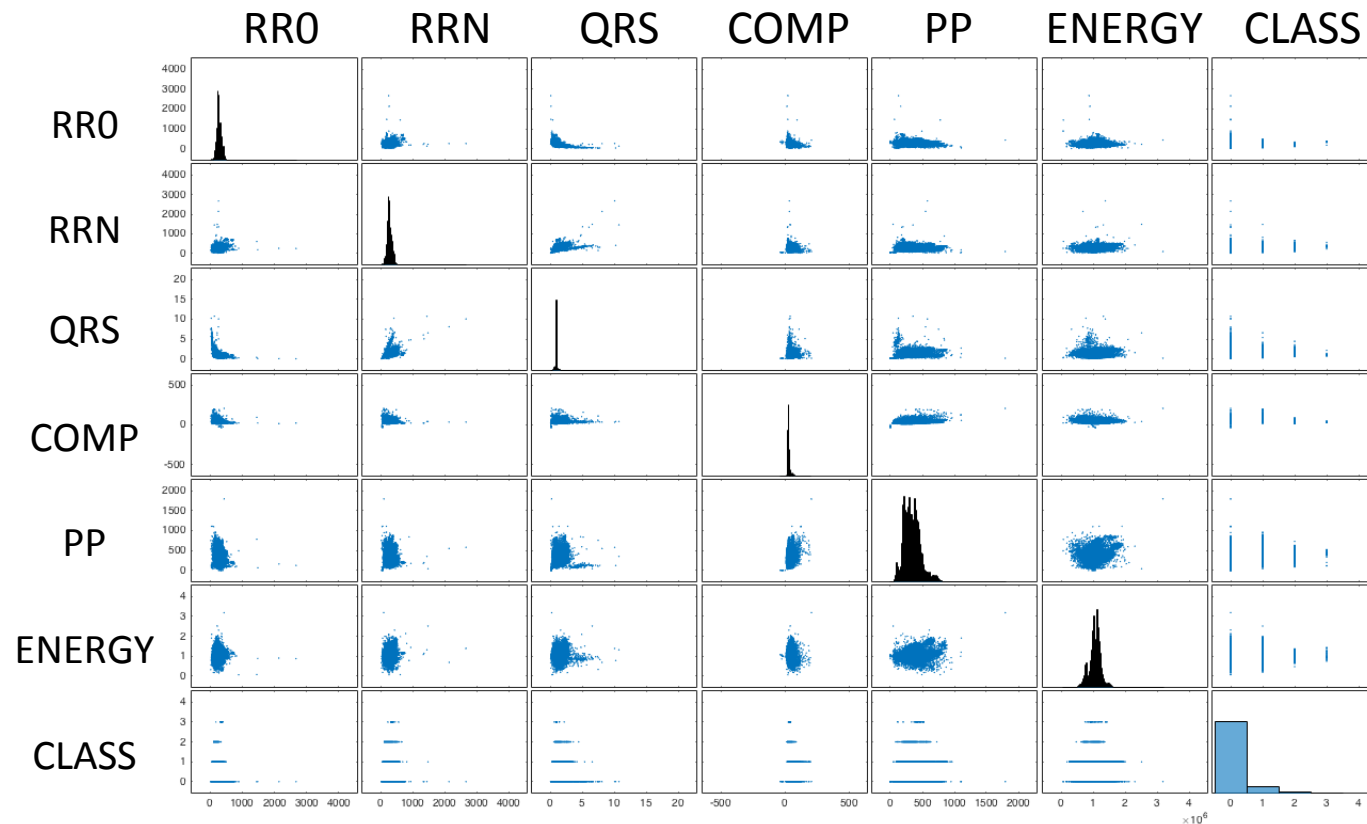


# Correlation

	RR0	RRN	QRS	COMP	PP	ENERGY	CLASS
RR0	1	49.7%	43.9%	28.4%	3.7%	9.1%	36.6%
RRN	49.7%	1	41.5%	1.9%	5.7%	3.5%	4.7%
QRS	43.9%	41.5%	1	33.0%	7.6%	6.1%	<b>41.1%</b>
COMP	28.4%	1.9%	33.0%	1	17.3%	5.5%	<b>49.4%</b>
PP	3.7%	5.7%	7.6%	17.3%	1	18.6%	13.4%
ENERGY	9.1%	3.5%	6.1%	5.5%	18.6%	1	0.3%
CLASS	36.6%	4.7%	<b>41.1%</b>	<b>49.4%</b>	13.4%	0.3%	1

- QRS and COMP exhibit maximum relationships within the corresponding classes.
- This confirms that QRS and COMP are very important in arrhythmias detection.

# Correlation



The histograms indicate the presence of a linear correlation.

# Features

- ***RR0***: *R-R* interval of the beat
  - the difference between the QRS peak of the present and previous beat
- ***RRS***: ratio *RR1-to-RR0*
  - the ratio of the present over the previous R-R interval
- **QRS width**
  - calculated according to the Tompkins algorithm
- Each beat is represented as 3-dimensional vector.

# Multinomial Logistic Regression (MLR)

- a supervised-learning algorithm
- a classifier able to distinguish among  $K$  classes
- Inputs the feature vectors of  $L$  labelled training samples:  $D_L = \{(X_1, Y_1), \dots, (X_L, Y_L)\}$ , which is called the training set.
- Computes the posterior class distribution using to estimate regression coefficients  $w$ .

# Logistic Regression – Training phase

The general MLR model is computed as:

$$P(y_1 = k | x_i, w) = \frac{\exp(w^{(k)} x_i)}{\sum_{k=1}^K \exp(w^{(k)} x_i)}$$

- $w$  is defined as  $(w^{(1)}, \dots, w^{(K-1)})$
- $w^{(k)}$  is the set of logistic regression coefficient for class  $k$
- $x = (x_1, \dots, x_i)$  are the feature vectors of training samples.



# Logistic Regression - Kernel

- A Gaussian Radial Basis Function (RBF) is defined as:

$$K(x_i, x_j) = \exp \left( \frac{-||x_i - x_j||^2}{2\sigma^2} \right)$$

- It describes the training vectors and offers improved data separability in the transformed space.

# Logistic Regression – Training phase

- The posterior probability density of  $w$  with
  - $Y_L$ : set of labels
  - $X_L$ : set of vectors of labelled samples

$$P(w|Y_L, X_L) \propto p(Y_L|X_L, w)p(w|X_L)$$

# Logistic Regression - Coefficients

Expectation Maximization (EM) is used to estimate the regression coefficients:

$$\hat{w} = \arg \max(l(w) + \log p(w|X_L))$$

where the log-likelihood function of  $w$  is computed as:

$$\begin{aligned} l(w) &\equiv \log p(Y_L|X_L, w) \equiv \log \prod_{i=1}^L P(y_i|x_i, w) \\ &\equiv \sum_{i=1}^L x_i^T w^{(y_i)} - \log \sum_{j=1}^K \exp \left( x_i^T w^{(j)} \right). \end{aligned}$$

# Logistic Regression – Test phase

- Regression coefficients ( $w$ ) are constant value inputs.
- Posterior class probability densities of each feature vector are computed.
- The class label of each feature vector is determined by the index of the maximum posterior class probability.

# Evaluation – Confusion Matrix

	<b>N</b>	<b>V</b>	<b>A</b>	<b>J</b>	<b>Other</b>
<b>N</b>	42016	7	9	3	0
<b>V</b>	16	3947	3	0	0
<b>A</b>	4	12	692	0	0
<b>J</b>	0	0	3	29	0
<b>Other</b>	0	0	0	1	0

- Columns represent heart beats in estimated classes, while rows represent beats in real classes.

# Evaluation

Metric	Score
Last Correct Rate	93.13%
Last Error Rate	6.87%
Inconclusive Rate	0.00%
Classified Rate	100.00%
Sensitivity	92.86%
Specificity	94.17%
Positive Predictive Value	81.25%
Negative Predictive Value	97.98%
Positive Likelihood	15.94%

# Evaluation: Parameters

- **CorrectRate:**  $\text{Correctly Classified Samples} / \text{Classified Samples}$
- **ErrorRate:**  $\text{Incorrectly Classified Samples} / \text{Classified Samples}$
- **LastCorrectRate:** CorrectRate computed only during the last results' update.
- **LastErrorRate:** ErrorRate computed only during the last results' update.
- **InconclusiveRate:**  $\text{Nonclassified Samples} / \text{Total Number of Samples}$
- **ClassifiedRate:**  $\text{Classified Samples} / \text{Total Number of Samples}$
- **Sensitivity:**  $\text{Correctly Classified Positive Samples} / \text{True Positive Samples}$
- **Specificity:**  $\text{Correctly Classified Negative Samples} / \text{True Negative Samples}$
- **PositivePredictiveValue:**  $\text{Correctly Classified Positive Samples} / \text{Positive Classified Samples}$
- **NegativePredictiveValue:**  $\text{Correctly Classified Negative Samples} / \text{Negative Classified Samples}$
- **PositiveLikelihood:**  $\text{Sensitivity} / (1 - \text{Specificity})$
- **NegativeLikelihood:**  $(1 - \text{Sensitivity}) / \text{Specificity}$
- **Prevalence:**  $\text{True Positive Samples} / \text{Total Number of Samples}$ .

# Conclusions

- Medical decision making requires automatic diagnostic support.
- Our method provides explicit knowledge base on class probabilities estimated on a medical database.
- MLR improves the transparency and interpretability of the classification process.
- In the future, we aim to integrate fuzzy partition rules into this method.



# Thank you! Any questions?

